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EXTREMES OF HYDROMETEORS AT ALTITUDE
FOR MIL-STD-210B. SUPPLEMENT: DROP-
SIZE DISTRIBUTIONS

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L. G. Hanscom Field, Massachusetts

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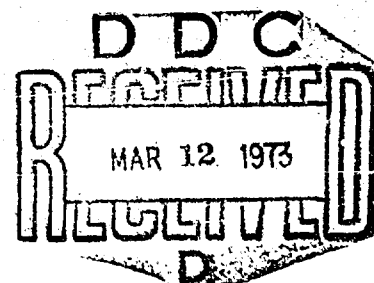
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Extremes of Hydrometeors at Altitude for MIL-STD-210B

Supplement — Drop-Size Distributions

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13. ABSTRACT

In the basic report for this supplement, estimates were provided for extreme rainfall intensities and associated liquid water content at altitude with 0.1 and 0.5 percent probabilities in the most severe month in the rainy tropics. Analogous values were also included for two all-time record rain storms: one for 1 min, and the other lasting 42 min. In this brief supplement, drop-size distributions are developed for the 1-min record, and the 0.1 and 0.5 percent "Worst Month Tropics" from a theoretical expression that best fits the available empirical size distributions. Liquid water content obtained from an integral of this theoretical expression is compared to the liquid water content obtained by arithmetically adding the volumes of each drop for three drop-size distributions.

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Extremes of Hydrometeors at Altitude
for MIL-STD-210B
Supplement —Drop-Size Distributions

1. INTRODUCTION

Sissenwine (1972), in the basic report for this supplement, provided estimates
of extreme rainfall intensities and associated liquid water content at altitude for
0.5 and 0.1 percent probabilities in the most severe month in the rainy tropics.
Analagous values were also included for the most severe all-time recorded
42-min and 1-min rain storms. These values were proposed for inclusion in
MIL-STD-210B, "Climatic Extremes for Military Equipment." At a subsequent
DoD Task Group meeting on this DoD document, a need for drop sizes and numbers
was expressed. In this brief addendum, drop-size distributions by altitude are
developed for the 1-min record (highest ever recorded) and the 0.1 and 0.5 per-
cent "Worst Month Tropics" profiles. Liquid water contents are computed for a
few of these drop-size distributions by arithmetically adding the volumes for
each drop. These are compared to estimates of liquid water content from an ex-
pression provided in the basic report which provides integrated water mass.

2. DROP-SIZE DISTRIBUTION

Dyer's (1969) review and summary of findings on drop-size distributions
serves as a basis for developing specific distributions herein. From an

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investigation of summer rain over Ottawa, Canada with intensities of 1 to 23 mm/hr, Marshall and Palmer (1948) found that except for small diameters, the size distribution of raindrops can be estimated using an exponential expression of the form

$$N_D = N_o e^{-\Lambda D} \tag{1}$$

where N_D is the number of drops of unit size range per unit volume, D is the drop diameter, N_o is a fictitious value of N_D, the intercept with the ordinate when D is zero (empirical studies indicate a cutoff in raindrop size between 0 and 0.5 mm), and Λ is a variable which is dependent upon the intensity of the precipitation. The relationship of Λ with rain intensity is fixed by the expression

$$\Lambda = aR^b \tag{2}$$

where R is the intensity and a and b are constants. These values vary with the investigator. Dyer also indicates that the size of the median volume raindrops (that is, the diameter that divides drops of larger and smaller diameter into groups of equal total volume), D_o, can be expressed in millimeters by

$$D_o = \frac{3.67}{\Lambda} \tag{3}$$

where Λ (mm^{-1}) is dependent on the precipitation intensity, R (mm/hr).

Several authors have derived values for N_o and Λ from empirical data for various locations and precipitation characteristics. Dyer (1969) summarizes the results of most of these. Since concern herein is with high intensity rainfalls, it would seem logical to use a distribution derived from thunderstorm data, but there is some diversity among investigators of drop-size distributions on the expression that best fits such distributions.

Mueller and Sims (1970) have recently presented empirical spectrums of drop-size distributions near the surface which were obtained by photographic observations for various rainfall intensities in the tropics, including higher rates of fall of greatest importance in military design. These are given in Figure 1. To this figure, theoretical drop-size distributions have been added for a rainfall intensity of 48 mm/hr. These were computed from Eq. (1) for N_o and Λ values suggested by three investigators:

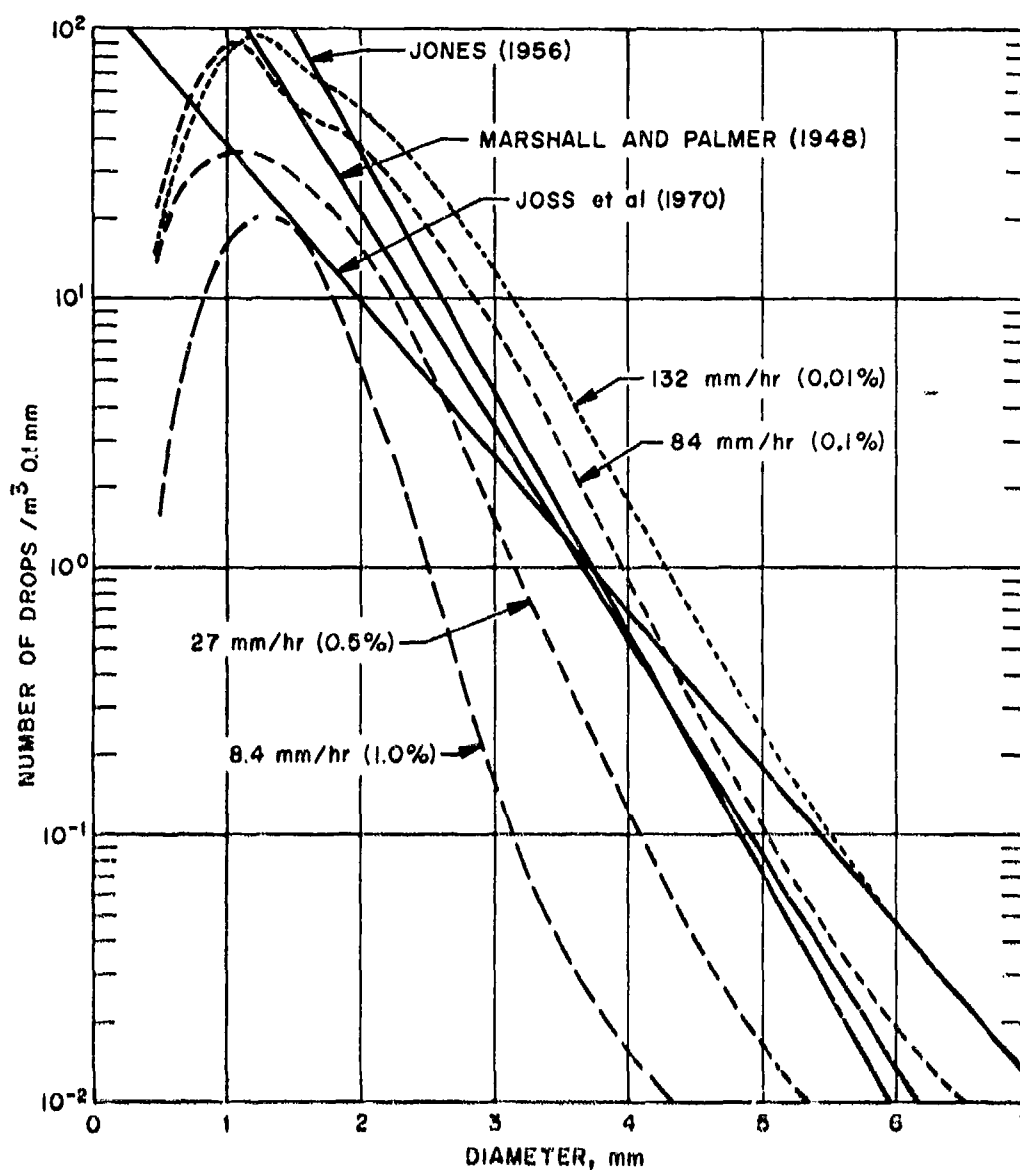| Source	Type of Precipitation	N_o (mm^{-1}m^{-3})	Λ (mm^{-1})	D_o (mm)
Marshall and Palmer (1948)	Ottawa, Summer	8,000	$4.1R^{-0.21}$	$0.9R^{0.21}$
Jones (1956)	Thunderstorm	$389R^{1.02}$	$2.5R^{-0.05}$	$1.48R^{0.05}$
Joss et al (1970)	Thunderstorm	1,400	$3.0R^{-0.21}$	$1.22R^{0.21}$

Figure 1. Average Drop-size Spectra for Natural Rainfall Rates Occurring 0.01, 0.1, 0.5, and 1.0 Percent of the Time (Dashed Lines). These curves are for tropical climates (Mueller and Sims, 1970). The theoretical distributions, continuous lines, for the three sources shown were computed for a precipitation rate of 48 mm/hr

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Computations were made for each 1-mm class interval starting with 0. 5 to 1.4 mm. An examination of Figure 1 indicates that the distributions provided by the first two investigators conform best to Mueller and Sims' data. The Joss distribution tends toward an unrealistically high number of very large drops while providing a relatively low estimate of the number of small drops. The straight lines of the theoretical distributions of Eq. (1) do not agree with curves of the empirical data below 1.5 to 2.0 mm diameter. It is apparent, however, that Eq. (1) overestimates the number of smaller drops since it indicates the greatest number when the diameter is zero.

The median size raindrops, D_o, for rainfall rates of 48, 188, and 1860 mm/hr are:

	Intensity (mm/hr)		
	48 (mm)	188 (mm)	1860 (mm)
Marshall and Palmer (1948)	2.03	2.70	4.37
Jones (1956)	1.80	1.92	2.16
Joss et al (1970)	2.80	3.66	5.93

Comparing these, the Jones (1956) medians are least sensitive to rain intensity, remaining close to 2 mm. This would indicate that a heavier rainfall has more drops of all sizes which is in agreement with the curves of empirical distributions of Mueller and Sims (1970) in Figure 1, except for a minor discrepancy in the two heaviest intensities for sizes below about 1.1 mm. The other two expressions provide unrealistically large medians at the higher rates. Figure 1 shows that Jones' values for N_o and Λ are most suitable for the intense rainfall rates with which we are concerned, and utilizing them is consistent with the basic report. The resulting equation for the distribution of drop sizes is

$$N_D = 389R^{1.02} \exp \frac{-3.67D}{D_o} \tag{4}$$

where

$$D_o = 1.48R^{0.05} . \tag{5}$$

Equation (4) can be integrated to express the total precipitation water content in the form of the equation

$$M = 0.052R^{0.97} \tag{6}$$

where M is the water content (g/m^3) and R the intensity (mm/hr). Sissenwine (1972) utilized this equation in developing his precipitation profiles.

Sissenwine (1972) considers the precipitation in the extreme storms presented in the basic study to be all liquid below 4.5 km. Between 4.5 and 10 km these storms contain ice and liquid particles mixed, becoming mostly solid (snow) above 10 km. Diameters of solid particles can be computed with equations presented by Dyer (1969) which are somewhat more complex than for liquid precipitation. The hypothetical aspects of applying these equations to extreme altitudes and intensities beyond that from which they were developed, however, does not warrant this additional refinement. Therefore, equivalent liquid water using Eqs. (4) and (5) will be assumed for higher altitudes.

Precipitation rates at the surface, 1 km, 2 km, and higher 2-km intervals up to 18 km were interpolated for the proposed MIL-STD-210B from Table 3 of Sissenwine (1972). Median diameters and the distributions of drop diameters, computed for these levels with Eqs. (4) and (5), are shown in Tables 1, 2 and 3.

To obtain the drop-size distributions for 1-mm intervals in Tables 1, 2 and 3, the midvalue for each millimeter class interval of the range of drop diameters was used for D (for example, 1 mm was used for D to compute the number of drops in the 0.50 to 1.49-mm class interval, etc.).

As already noted, Eq. 4 indicates that as the drop size D approaches zero, the number of drops continues to increase toward $339R^{1.02}$. This is unrealistic, since it leads to the largest number of drops at zero diameter. This is a result of fitting the distribution of raindrop sizes with a simple equation which neglects physical limitations that govern both large and small sizes attainable in nature. The most important contribution to the total water content, however, comes from the center of the distribution at diameters of 2 to about 5 mm. The assumption that the logarithm of the number of drops varies inversely with the size, as in Eq. (4), is borne out by Figure 1. Figure 1, however, also shows that this theoretical distribution departs from the shape of the empirical curves at diameters less than 2 mm when Jones' (1956) values for N_o and Λ are used. This shortcoming should not be of great importance to the total liquid water content since the difference in water mass for these very small drops is small.

Another important input from Figure 1 is that the empirical data, in addition to showing that the number of drops has a maximum at sizes of 1 to 1.5 mm depending upon intensity, also indicates there are no drops of less than roughly 0.5-mm diameter. A sharp cutoff at 0.5 mm is questionable, but the relatively slower terminal velocity of the smallest drops will lead to collision and coalescence with faster falling larger drops, providing a sharp diminution of drops at some diameter evidently not too much lower than 0.5 mm. For this reason, the smallest size class used for the distributions provided in Tables 1, 2 and 3 is 0.5 to 1.4 mm.

Table 1. Number of Drops (Liquid or Equivalent Liquid) per m^3 for the 1-min World-Record Rainfall

Altitude (km)	Rainfall Intensity (mm/min)	Median Diameter (mm)	Diameter (mm)					
			0.5-1.4	1.5-2.4	2.5-3.4	3.5-4.4	4.5-5.4	5.5-6.4
0	31	2.2	158,624	29,915	5642	1064	201	38
1	34	2.2	174,297	32,870	6199	1169	220	42
2	36	2.2	184,761	34,844	6571	1239	234	44
4	41	2.2	210,970	39,787	7503	1415	267	50
6	42	2.2	216,220	40,777	7690	1450	274	52
8	30	2.2	153,407	28,930	5456	1029	194	37
10	20	2.1	93,699	16,321	2843	495	86	15
12	14	2.1	65,123	11,344	1976	344	60	10
14	9	2.0	38,024	6,069	969	155	25	4
16	4	1.9	15,097	2,188	317	46	7	1
18	1	1.8	3,298	429	56	7	1	< 1
20	0	-	-	-	-	-	-	-

Table 2. Number of Drops (Liquid or Equivalent Liquid) per m^3 Equalled or Exceeded With 0.1 Percent Probability, Worst Month, Wet Tropics

Altitude (km)	Rainfall Intensity (mm/min)	Median Diameter (mm)	Diameter (mm)					
			0.5-1.4	1.5-2.4	2.5-3.4	3.5-4.4	4.5-5.4	5.5-6.4
0	3.13	1.9	11,755	1,704	247	36	5	1
1	3.38	1.9	12,714	1,843	267	39	6	1
2	3.63	1.9	13,674	1,982	287	42	6	1
4	4.11	1.9	15,521	2,249	326	47	7	1
6	4.23	1.9	15,983	2,316	336	49	7	1
8	2.97	1.9	11,143	1,615	234	34	5	1
10	2.03	1.9	7,559	1,095	159	23	3	< 1
12	1.38	1.8	4,580	696	78	10	1	< 1
14	0.85	1.8	2,794	364	47	6	1	< 1
16	0.43	1.7	1,237	143	16	2	< 1	< 1
18	0.09	1.6	219	22	2	< 1	< 1	< 1
20	0	-	-	-	-	-	-	-

Table 3. Number of Drops (Liquid or Equivalent Liquid) per m^3 Equalled or Exceeded With 0.5 Percent Probability, Worst Month, Wet Tropics

Altitude (km)	Rainfall Intensity (mm/min)	Median Diameter (mm)	Diameter (mm)					
			0.5-1.4	1.5-2.4	2.5-3.4	3.5-4.4	4.5-5.4	5.5-6.4
0	0.80	1.8	2626	342	45	6	1	< 1
1	0.87	1.8	2861	372	48	6	1	< 1
2	0.93	1.8	3062	399	52	7	1	< 1
4	1.0	1.8	3298	429	56	7	1	< 1
6	1.1	1.8	3634	473	62	8	1	< 1
8	0.77	1.8	2526	329	43	6	1	< 1
10	0.51	1.8	1659	216	28	4	< 1	< 1
12	0.35	1.7	1002	116	13	2	< 1	< 1
14	0.22	1.7	624	72	8	1	< 1	< 1
16	0.11	1.6	269	27	3	< 1	< 1	< 1
18	0.02	1.5	41	4	< 1	< 1	< 1	< 1
20	0	-	-	-	-	-	-	-

The dashed curves in Figure 1 are based upon Panama data only; therefore, probability percentages on each curve represent only that area of the moist tropics. Intensities for these probabilities are lower than those for world-wide rainier tropical areas derived for MIL-STD-210B by Salmela et al (1971) and used by Sissenwine (1972). Comparison of the MIL-STD-210B 1.0, 0.5, 0.1 percent and world-record intensities for the rainiest month to Panama intensities (mm/hr) specified in Figure 1 follows:

| | Probability (%) | | | | |
	1.0	0.5	0.1	0.001	World Record
Panama	8.4	37	84	132	----
MIL-STD-210B	30	48	188	---	1872

A 0.001 percent world-wide worst month value was not determined for MIL-STD-210B.

Drop numbers in Tables 1, 2 and 3 are provided for diameters up to 6.4 mm, but drop diameters greater than 5.5 mm are unstable according to Fletcher (1962). Curves in Figure 1, however, fail to indicate a sharp drop in numbers of the very large sizes. This discrepancy is probably a result of an inadequate sample size.

3. LIQUID WATER CONTENT

Several equations have been developed, mostly by radar meteorologists, for relating liquid water content to precipitation intensity. Four of these are presented by Sissenwine (1972). The relationship derived from Eq. (4), Jones (1956)—which Sissenwine found best supported in the scientific literature for these very severe thunderstorms—is Eq. (6). Approximations involving the cutoff of maximum and minimum drop sizes were made to obtain the drop-size distributions from Eqs. (4) and (5) for Tables 1, 2 and 3. The importance of these approximations can be estimated from a comparison of precipitation mass at the surface in the three storms of Tables 1, 2 and 3, obtained by adding the volume of the raindrops, to the values from Eq. (4):

| | Liquid Water Content | | |
	0.5% (g/m^3)	0.1% (g/m^3)	1-min World Record (g/m^3)
Equation (4)	2.2	8.3	77.2
Drop mass total	1.8	8.7	169.0

The values for the water mass from the drop total were obtained by multiplying the number of drops in a 0.1-mm diameter range by the volume of a drop with a diameter in the center of that range, and summing the total within the cutoff

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diameters; that is, 0.5 to 6.4 mm. The water mass obtained using this procedure should be less than the amount obtained using Eq. (6), since only a certain range of drop sizes was used in this computation whereas the entire spectrum of drop sizes is inherent in using Eq. (6). Since there is a logarithmic increase in the number of drops with decreasing diameter, however, there is a bias in our method of computing the water mass. The reason for the bias is our use of the diameter in the center of the range to compute the drop volume for all the drops in that range. The result is an overestimate of the drop volume with increasing rainfall rate. This is apparent in the above comparison especially for the 1-min world record. The discrepancy at the more realistic extreme rainfall intensities is small. With this in mind, it can be seen that the difference between water mass computed using Eq. (6) to that using the drop total is not of practical importance.

4. CONCLUSIONS

The distribution of drop sizes in Tables 2 and 3 gives reasonable numbers of drops for the spectrum of diameters in the "0.1 and 0.5 percent Worst Month Tropics" synthetic storms proposed for MIL-STD-210B (Sissenwine, 1972).

The 0.5 percent intensity storm was suggested as mandatory for operation of combat equipment. The values in Table 1 give very improbable all-time extremes that can be considered a "ball park" goal when life would be endangered by an encounter with exceptional extremes. The values in Table 2 are for as intense a storm as could be assigned a reasonably reliable probability. They certainly should be attained as design criteria for equipment designed for world-wide usage in which failure would endanger life.

Acknowledgments

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